

Nonlinear gauge interactions: a possible solution to the “measurement problem” in quantum mechanics

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Abstract

Two fundamental, and unsolved problems in physics are:

- i) the resolution of the “measurement problem” in quantum mechanics
 - ii) the quantization of strongly nonlinear (nonabelian) gauge theories.
- The aim of this paper is to suggest that these two problems might be linked, and that a mutual, simultaneous solution to both might exist.

We propose that the mechanism responsible for the “collapse of the wave function” in quantum mechanics is the nonlinearities already present in the theory via nonabelian gauge interactions. Unlike all other models of spontaneous collapse, our proposal is, to the best of our knowledge, the only one which does not introduce any new elements into the theory. A possible experimental test of the model would be to compare the coherence lengths - here defined as the distance over which quantum mechanical superposition is still valid - for, *e.g.*, electrons and photons in a double-slit experiment. The electrons should have a finite coherence length, while photons should have a much longer coherence length (in principle infinite, if gravity - a very weak effect indeed unless we approach the Planck scale - is ignored).

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1 Introduction

Simply stated, the “measurement problem” in quantum mechanics is that measuring instruments, and observers, are made up of quantum entities (atoms, etc) and, according to the Schrödinger equation, quantum mechanical superpositions never stop. How then, is a definite outcome of a measured “observable”, instead of the predicted quantum superposition, possible? That is, where, and how, is the superposition broken by a non-unitary “collapse”? The orthodox theory is completely silent on this point, and it is simply postulated that somewhere between observed (quantum) object and (classical) observer, the “collapse” takes place.

As a possible solution to this, we start by noting that:

- The measurement problem, or the collapse of the wavefunction in quantum mechanics is not solved.
- The quantization of *strongly* nonlinear gauge theories is not solved, *e.g.* strongly coupled Yang-Mills and gravity near the Planck-scale.

These statements can essentially be rephrased as:

- The absolute backbone of quantum mechanics is the *superposition principle* [1] (*e.g.*, interfering amplitudes, summation of Feynman diagrams, etc). It is also well known that superposition requires *linear* equations, *i.e.*, the sum of two different solutions to a nonlinear equation is generally *not* a solution, ruining superposition. The Hilbert space of quantum mechanics and the Fock space of quantum field theory are *linear* spaces (based on the superposition requirement), suitable for linear mappings or operators.
- Nonabelian, *i.e.* noncommutative, gauge field theories describing the fundamental interactions obey *nonlinear* evolution equations in the gauge fields. As the gauge fields are supposed to be quantum mechanical this is in apparent contradiction to the previous statement. For convenience, we will write down the evolution equations for (pure) Yang-Mills fields below. Although the fermion evolution obeys linear equations, it becomes “contaminated” by nonlinearities through the interactions.

The nonabelian vector gauge fields, \mathbf{A} , are governed by a set of coupled, second order, nonlinear PDEs on Minkowski spacetime. (The general

argument for gravity is similar [2], but involve tensor fields constituting a dynamical spacetime. We do not explicitly write down those equations.) For pure Yang-Mills the evolution equations are given by

$$(\partial^\mu - g[A^\mu, \cdot])_a^b (\partial_\mu A_\nu - \partial_\nu A_\mu - g[A_\mu, A_\nu])_b = 0, \quad (1)$$

where g is the coupling constant and a, b are indices of the gauge group, *e.g.*, $a, b \in 1, 2, 3$ for $SU(2)$ and $a, b \in 1, \dots, 8$ for $SU(3)$. Summation over repeated indices is implied. The operator (“covariant derivative”) at the left works according to $(\partial^\mu - g[A^\mu, \cdot])(anything) = \partial^\mu(anything) - g[A^\mu, anything]$. We see that we get highly nonlinear (quadratic and cubic) terms in the gauge fields, especially when the coupling constant, g , is large. The commutator terms, “square brackets”, vanish identically for abelian fields (*e.g.* photons) as the gauge fields then commute, leaving only the ordinary, linear Maxwell equations.

In the Feynman path-integral formulation of quantum mechanics [3] the nonlinearities can be “hidden” in the action functional, but as the Schrödinger, Heisenberg and path-integral formulations are equivalent, a problem in one of them must translate into a problem in all.

That quantization of nonabelian theories is troublesome, and hitherto unsolved in the strong field limit, or equivalently in the large coupling constant limit, arises from their nonlinear form. (One example of this, among many, is the “Gribov ambiguity” [4], which implies non-uniqueness of gauge transformations. This effect is absent in perturbation theory.) The “ghost” fields needed to preserve unitarity in nonabelian quantum field theory might be seen as a consequence of trying to constrain the truly nonlinear theory into a mildly nonlinear theory defined by Feynman diagrams.

Also, the intuitively simple picture of elementary particles (quanta!), which arises from perturbation theory, *e.g.* through Feynman diagrams, is absent in the strongly coupled regime, and if we assume that “quantization” is equivalent to the exchange of elementary quanta, it is obvious that quantum mechanics somehow must fail in the strong regime.

2 Idea

Instead of trying to solve these two problems separately, we propose that they are, in fact, related. We assume that instead of quantizing strongly nonlinear theories, it is their actual nonlinearities which are responsible for

breaking quantum mechanical superposition, hence “turning a vice into a virtue”.

We thus get a *self-induced collapse* into the ordinary world of chairs, tables, people and indeed also recorded elementary particle tracks in a photographic emulsion, a bubble chamber, or a modern multi-purpose computer-aided detector.

It should be stated that this is quite the opposite of what is normally considered in so called “quantum chaos”. There one studies what consequences a chaotic classical system has for the corresponding (chaos suppressing) quantum system, especially in the semi-classical, short wavelength regime. We, on the other hand, propose that the nonlinearity in the underlying dynamics could be responsible for the seemingly random character of quantum mechanics. This is possible as it is well known that classical gauge field theories can be chaotic [5] (in which case no analytic, closed formula solutions exist, seemingly precluding any simple “quantization recipe”). In “usual”, non-relativistic quantum chaos the nonlinearities would merely introduce higher order terms in the operator potentials in the Hamiltonian. However, it should be remembered that non-relativistic quantum mechanics is but an approximation to relativistic quantum mechanics, which itself is an approximation to quantum field theory. In quantum field theory the conceptual difference between fermions (“matter”, ψ) and bosons (“interactions”) almost disappears. *Both* fermions and bosons are now operators, differing only in that the former obey anti-commutation relations, while the latter obey commutation relations. Hence ψ is no longer privileged as is the case in non-relativistic quantum mechanics.

Also, from an intuitive physical viewpoint, it is obvious that nonabelian gauge bosons cannot behave like their abelian counterparts. In the Young double-slit experiment, a photon can “interfere with itself” as a route through one of the slits superposes with a route through the other slit (actually there are infinitely many superpositions), resulting in the familiar interference pattern. It is clear that this does no longer hold true if the photon is replaced by a nonabelian gauge boson, as the evolution equations now are nonlinear, destroying the general superposition possibility. The reason is that the photons can be represented by harmonic oscillators [6], while nonabelian fields cannot (also casting doubt on the existence of nonabelian quanta). For example, in a hypothetical world built by “color” alone, which is not completely absurd, as “glueball”-states consisting solely of color fields are expected to exist, the state vector would be constructed from the color potentials, but as these

are described by eight coupled, nonlinear PDEs, the superposition principle would be lost.

The mathematical formalism underlying all this can be summarized as follows:

For abelian theories, such as QED, a solution to equation (1) can be found by the usual superposition of Fourier analysis (dropping the normalization factor),

$$A_{\mu}^{Abelian} = \int d^3k \sum_{\lambda=0}^3 [a_k(\lambda) \epsilon_{\mu}(k, \lambda) e^{-ik \cdot x} + a_k^{\dagger}(\lambda) \epsilon_{\mu}^*(k, \lambda) e^{ik \cdot x}], \quad (2)$$

where ϵ_{μ} is the polarization vector. For a nonabelian theory this is no longer true,

$$A_{\mu}^{bNonabelian} \neq \int d^3k \sum_{\lambda=0}^3 [a_k^b(\lambda) \epsilon_{\mu}(k, \lambda) e^{-ik \cdot x} + a_k^{b\dagger}(\lambda) \epsilon_{\mu}^*(k, \lambda) e^{ik \cdot x}], \quad (3)$$

as Fourier methods are inapplicable to nonlinear PDEs (see , *e.g.* [7]). This signals the breakdown of quantum mechanical superposition for nonlinear (nonabelian) gauge fields. As the Fourier coefficients, (a_k, a_k^{\dagger}) , gives the connection to the gauge quanta through the corresponding annihilation and creation operators, the absence of the above expansion in the nonabelian case puts nonabelian quanta, and hence adherence to at least naive quantum mechanics, in doubt.

That a quantum mechanical state must be able to “self-collapse” in some way is imperative to obtain a “classical world” of macroscopic objects, and especially so in quantum cosmology, the quantum mechanical treatment of the whole universe, where no “outside” observer exists. The self-induced collapse puts an end to the infinite regress of quantum superposition, where first the measuring apparatus obtains a quantum mechanical nature, then the observer, and so on, ad infinitum, until the whole universe consists of infinitely many superimposed quantum states, without any one of them actually being “realized”. The “many worlds” interpretation of Everett [8] purports to solve this problem, without collapse, by assuming that we only see events which take place in one of these branching universes, but it seems that the fundamental question of *when*, and how, the universe actually branches is unanswered by that model, this being the equivalent of the “measurement problem” in the orthodox interpretation. There also seems to exist some empirical results pointing in the direction of self-collapse. For instance, in

[9], the experimental evidence of a non-zero (saturation) decoherence rate for electrons, even at zero temperature, seems clear. Thus quantum mechanical particles ought to be able to decohere *intrinsically*, without any influence from the environment [10] (*e.g.*, “heat bath”).

3 Implementation (rough)

We now turn to a preliminary, admittedly rough implementation of our idea of self-induced collapse. Following Penrose [2], we choose the following expression for the self-collapse time

$$\tau = \frac{\hbar}{E_{N.L.}}, \quad (4)$$

where $E_{N.L.}$ is the energy stored in the nonlinear field configuration of the nonabelian interaction (which in turn depends on the strength of the coupling). This choice is also compatible with the experimentally observed saturation temperature in e - e interaction corrections for conductivity [9]. Observe that the relation is *not* an uncertainty relation, despite its identical form, as τ and $E_{N.L.}$ are not uncertainties. We see that the relation gives the right classical “correspondence” limit when $\hbar \rightarrow 0$. The build-up and collapse of the field configuration could follow a pattern similar to the collapse of other nonlinear waves [11], making $E_{N.L.}$, and hence τ , effectively random, in agreement with radioactive decay, etc.

As we want the energy of the full nonlinear theory, but cannot today calculate this inherently non-perturbative quantity exactly, we take it to be a characteristic energy for the interaction. If, for instance for QCD, we as a rough approximation take the energy to be $E_{N.L.} \sim \Lambda_{QCD} \sim 0.2$ GeV, we get a “ballpark” figure of $\tau_{QCD} \sim 10^{-23}$ s for the collapse time for *strong* QCD, *e.g.*, inside a non-disturbed hadron. Although the exact result for τ_{QCD} might differ by many orders of magnitude, this may help explain why (semi-)classical models work so well for strong QCD, as the stronger the interaction is, the more “classical” it behaves according to our mechanism.

In our model, the quantum mechanical (linear, unitary) evolution is constantly punctuated by “hits” of self-collapse at an average frequency of $\langle \tau \rangle^{-1}$. This is similar to the case in orthodox quantum mechanics where an observation, or the initial preparation of a state, suddenly “realizes” one of the potential outcomes, after which the unitary (linear) evolution of the state takes

over until the next observation. A “macroscopic” piece of matter has such a huge energy stored in nonlinear field configurations that $\tau = \hbar/E_{N.L.} \sim 0$, approximating a continuously collapsing state, *i.e.*, a classical state. This forbids quantum mechanical effects to “invade” the macroscopic world, and resolves the “Schrödinger’s cat” paradox [12] and related questions such as “Wigner’s friend” [13], etc.

Note that *any* significant nonlinear interaction, whether as part of a “measurement” carried out by conscious beings [12, 13] or not, bring about the collapse of interfering amplitudes into classical states. (In contrast, Wigner [13] postulated that grossly nonlinear equations of motion should replace ordinary quantum mechanical evolution for conscious beings *only*.) Conscious observation is therefore only a *special case* of the more general nonlinearity, as all “measuring apparatuses”, including human beings, consist of both weakly (all constituents) and strongly (quarks) nonlinearly interacting fields. Hence, there should be no need to introduce the *mind* into the interpretation of quantum mechanics at a fundamental level.

If we would consider just pure QED the nonlinear terms would be absent, hence a hypothetical world built by QED alone would never be classical. This also explains why, *e.g.*, atomic physics, and the numerous recent tests of quantum mechanics using laser setups, works so closely to orthodox quantum mechanics (before “measurement”), as it is being “classicalized” only by (very) weak interaction effects. Were it not for the existence of other interactions besides QED, we would indeed have quantum mechanical superpositions of whole universes, *i.e.*, the “many worlds” interpretation of quantum mechanics by Everett [8].

One difference between our proposal for self-induced collapse, and other models aiming at the same goal, is that, as far as we know, all other models postulate additional equations and/or variables,

- *Decohering histories* [14, 15]: new fundamental principle of irreversible coarse graining + additional constraints to remove “too many” decoherent histories, also, superpositions are not really removed [16] (different outcomes still “coexist” as a mixture after decoherence) so “measurements” could be undone
- *Altered Schrödinger equation* [17, 18]: obvious extra (nonlinear) term added
- *Bohm QM* [19]: additional (nonlinear) evolution equation for objective

particle positions

whereas we use only nonlinearities which are *already present* in the dynamics of the accepted standard model of particle physics. Another difference is that, to our knowledge, all other models for spontaneous collapse/decoherence are non-relativistic, whereas our scheme is based on covariant theories. This also opens up for a treatment of “measurement” in quantum field theory.

As a nonlinear gauge evolution is effectively non-reversible, especially if chaotic [5], it lies close to identify it with the physical “mechanism” of the “irreversible amplification” emphasized by Bohr as being necessary to produce classical, observable results from the quantum mechanical formalism. Even though Bohr himself denounced the need, or even the possibility, to give a physical description of this “mechanism” [20], we believe that the central issue for truly understanding quantum mechanics lies in the quantum measurement problem. For instance, it is *only* there, in the collapse of the wave function (or its equivalent), that the indeterminacy of quantum mechanics enters. It is simply impossible to obtain the observed random behavior from the linear Schrödinger equation alone, without additional postulates or assumptions. It may, however, be possible that deterministic chaos in the nonlinear self-interaction can be responsible for the seemingly stochastic character of quantum mechanics, by collapsing to different alternatives effectively at random. In a way, it would even be surprising if quantum mechanics *is* fundamentally probabilistic, as in all other cases probabilities are *derived* from an underlying, deterministic dynamics. It may even be argued that the statistical patterns that arises in quantum mechanics may be taken as implicit proof that an underlying structure is present, as pure randomness would give *no* statistical correlations, contradicted by, *e.g.*, the observed definite average lifetimes for radioactive nuclei or the distinctive pattern arising in a two-slit experiment. At least one *local*, deterministic model of this sort has already been constructed [21] in a somewhat different context (utilizing gravity, and using “riddled”, or intertwined, chaotic attractor basins to sidestep Bell’s theorem [22]).

Our model can be experimentally tested, at least in principle, as differently charged (electric, weak, color,...) fields should have different coherence lengths. In a double-slit experiment, for instance, the photon should have a much longer (in principle infinite if gravity is ignored) coherence length than, *e.g.*, electrons which ought to have a finite coherence length due to nonlinear weak interactions. As the full nonlinear calculations are very complex, it is

not possible to quantitatively predict the coherence lengths at the present time, but if it turns out that electrons experimentally have shorter coherence lengths than photons it would strengthen our hypothesis¹. As another example of a possible experimental test, the coherent quantum state of, *e.g.*, the huge number of electrons in a superconducting ring should slowly decohere by the weak interaction, destroying the quantum mechanically superposed magnetic field configurations.

Given that our model is based on somewhat tentative and unfamiliar concepts, the fact that it should be amenable to experimental tests of this sort might still give it some credibility. Many of the other models proposed for solving the measurement problem predict experimental results identical to orthodox quantum mechanics, hence making them more interpretive than physical.

The collapse usually postulated in quantum mechanics is not relativistically covariant, as it is instantaneous over all space. Only the *deterministic* unitary development of the state is taken into account by the relativistic Dirac equation and, more generally, by quantum field theory. As our scheme for collapse is based on covariant gauge-field theories, it might be possible to describe the collapse of the state in a covariant way, although our present rough attempt, which for *simplicity* singles out the energy accumulated in the nonlinear field configurations, is not covariant. On the other hand, and perhaps more plausible, it can be argued that the collapse should be described by an inherently non-local mechanism, as it seems that quantum mechanics (and nature) at its very foundation *is* non-local, as given by the results of Aspect *et al.* [23], and more recent experiments on quantum non-separability. As a nonlinear theory also in some cases can be non-local, it would be interesting to investigate if this view of spontaneous collapse can account for these crucial non-local effects. Work in this direction is in progress [24], together with a more detailed investigation of the nonlinear terms in the nonabelian evolution equations, as a means to better understand the quantitative details of the proposed mechanism for self-collapse.

¹One referee asked about observed interference effects in double-slit experiments using neutrons and C_{60} . The results should, according to the ideas presented here, be $\tau_\gamma > \tau_e > \tau_n > \tau_{C_{60}}$ due to $\infty = \tau_{\text{pureQED}} > \tau_{\text{weakint}} > \tau_{\text{residualQCD}} > \tau_{\text{multinucleon}}$. This does not contradict present observational data.

4 Conclusion

The gist of our proposal can be illustrated by an analogy:

i) Do ocean waves collapse (“break”) only when they are observed? No, they automatically collapse as dictated by their nonlinear dynamics.

ii) Do quantum mechanical waves collapse only when observed? No, they automatically collapse as dictated by the nonlinear dynamics in their interactions (gauge fields).

The analogy admittedly halts a little, as quantum mechanics “lives” in an abstract, multidimensional configuration space.

However, it is a fact that

- no consistent solution to the quantization of strongly nonlinear theories yet exists
- nonlinear terms break quantum mechanical superposition

Maybe nature is trying to tell us that strongly nonlinear theories should *not* be quantized, and that they instead are responsible for turning the quantum “potentialities” into classical “actualities”?

We have described how automatic dynamical collapse of the wave function in quantum mechanics may already be implicit in the existing dynamical theory, that of nonabelian gauge fields. These include the weak interaction, QCD, gravity, and any other nonabelian fields which eventually may be found in the future. The nonlinear self-interaction terms break the fundamental superposition principle of quantum mechanics, inducing just the right physical mechanism for the purpose of solving the “measurement problem”.

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